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A homotopy method for determining the eigenvalues of locally or non-locally reacting acoustic liners in flow ducts

Xiaofeng Sun[a,*], Lin Du[a], Vigor Yang[b]

[a]School of Jet Propulsion, Beijing University of Aeronautics and Astronautics, Beijing, China
[b]Department of Mechanical Engineering, The Pennsylvania State University, PA 16802, USA

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Abstract

This paper presents a unified algorithm for studying the eigenvalue problem of a lined duct using the homotopy method. Results from various validations show that the method developed in this work can provide accurate and reliable numerical solutions for sound-propagation computation. The investigation also indicates that homotopy methods not only overcome the computational difficulties of the existing methods for locally reacting liners, but also give a completely different way to calculate the eigenvalues of non-locally reacting liners, which have recently received considerable attention due to their potential application for future advanced liners. Finally, a model multi-segmented, non-locally reacting liner is employed to study the possibility of controlling sound attenuation through a bias flow. The simulation shows that by adjusting the bias flow of each segment, optimal sound attenuation can theoretically be achieved.

1. Introduction

Acoustic liners are used extensively to suppress sound propagation in ducts. The design of these liners to achieve the most efficient attenuation has been a major subject of expert concern. For this reason, various methods for predicting the attenuation of acoustic waves in a lined duct have been developed over the last several decades. One of these methods is based on solving the eigenvalue equation in order to compute the radial wavenumbers for each duct propagation mode. The solution of eigenvalues is hence critical for this method to work robustly and efficiently. The primary objective of the present work is to make use of homotopy methods to form a unified algorithm, so as to study the eigenvalue problem appearing in both locally and non-locally reacting liners and also achieve insight into the mechanism of the interaction of sound propagation with various sound-absorption structures.

Locally reacting liners that permit propagation only in the direction normal to the duct wall usually consist of a perforated sheet or thin layer of porous material followed by a honeycomb array and backed by the impervious wall of the duct. In general, we can describe the wall-boundary condition with the help of the concept of acoustic impedance without directly considering the sound propagation inside the liners.

*Corresponding author: Tel./fax: +86 10 82317408.
E-mail address: sunxf@buaa.edu.cn (X. Sun).

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The eigenvalue equation derived from the impedance condition is usually solved by Eversman's integration method [1,2], in which a nonlinear ordinary differential equation (ODE) is derived by introducing a parameter perturbation to the transcendental algebraic equation. The solution of eigenvalues is thus changed into an initial-valued problem. With this method, one can obtain the eigenvalues by numerically solving the ODE with the hard wall eigenvalues as the initial condition of the soft wall. This method has had a great influence on the study of sound propagation in ducts with locally reacting liners, especially in the acoustic design of aircraft nacelles. There is singularity, however, at the lowest-order mode, the plane wave, in the ODE derived by Eversman's method for both rectangular and circular ducts [3]. Furthermore, some investigations have also shown that there are numerical instabilities or mode jump phenomena under some impedance boundary conditions. The problem has received little attention in recent years, although it remains insufficiently understood. It is noted, however, that the calculation of the eigenvalues still plays an important role in the preliminary stage of duct acoustic design in many fields related to practical engineering applications. Overcoming the singularity problem apparent in the existing method by developing a new approach is also a problem of academic interest. Non-locally reacting liners, also known as bulk-reacting liners, permit propagation in more than one direction. Their applicability to a relatively wide frequency range and ability to control sound attenuation has received considerable attention in both experimental and theoretical investigations [4–10]. Especially for the calculation of the eigenvalues of non-locally reacting liners, most of the work is based on the Newton–Raphson scheme. Common problems are the selection of initial values and the convergence of the algorithm. Once again, a method for accurately and effectively solving the eigenvalues still deserves to be further investigated for both locally and non-locally reacting liners.

The homotopy method may effectively resolve the problem of computing eigenvalues. The method, as first proposed by Scarf [11] and further developed by Eaves and Scarf [12], offers a powerful means of determining solutions to complex systems of equations [13]. In this paper, we will use the basic concept of the homotopy method to constitute various homotopy equations to calculate the eigenvalues for both locally and non-locally reacting liners. In this framework, the homotopy equation we suggest has no singularity for the lowest mode, i.e., the plane wave mode, and one can flexibly choose homotopy parameters to avoid numerical instability or mode jump problems. In particular, for a non-locally reacting liner, the effect of the liner cannot be described by effective impedance, since acoustic waves in the liner also propagate parallel to the wall. Because of this, it is invalid to directly extend Eversman's method to the non-locally reacting liners. However, with the application of the homotopy concept, it is possible to constitute the homotopy equation for the non-locally reacting liners. It can then be changed into a nonlinear ODE and then solved as an initial problem of the ODE. We present various numerical results for both porous liners and perforated liners, with an emphasis on comparison with existing results. In particular, we compare our numerical results with the recent experimental data by Eldredge and Dowling [10], and show very good agreement. Finally, this work shows that it is convenient to extend the present model to include the effect of multi-segmented non-locally reacting liners, and is suitable for optimizing computation by making use of the existing transfer matrix or mode-matching method presented by Zorumski [14]. Our numerical simulation for multi-segmented liners shows the capability to control the sound attenuation with bias flow.

In the following sections, we will first make a brief introduction to the homotopy methods related to our research and then introduce various numerical results by constituting different homotopy equations for both locally and non-locally reacting liners. In the last part, we will suggest a model for studying multi-segmented non-locally reacting liners and then give the relevant numerical results and analysis.

2. Homotopy methods

Homotopy, or continuation, methods are globally convergent numerical techniques for solving nonlinear algebraic equations. They have been used extensively for various practical applications [13], such as trajectory generation and DC operating point problems. As mentioned in the introduction, we extend this method to the eigenvalue problem in duct acoustics, which has not been done previously. For this specific objective, it is necessary to make a brief description of homotopy methods, with an emphasis on the relevant homotopy formulation in connection with our eigenvalue algorithm. Suppose that we wish to find a solution

$x \in D \subset R^n$ to

$$F(x) = 0 \tag{1}$$

which we know to exist, where $F : R^n \to R^n$ is a smooth mapping, and D is compact. Without a good approximation of the zero point x, an iterative solution to Eq. (1) will often fail, because poor starting values are likely to be chosen. As a possible remedy, we might first solve an easier system, also having a solution in D

$$G(x) = 0, \tag{2}$$

where $G : R^n \to R^n$. From these, we can construct the linear homotopy

$$H(x,t) = tF(x) + (1-t)G(x), \tag{3}$$

where $H : R^{n+1} \to R^n$. For $t = 0$, $F(x,0) = G(x) = 0$ is a vector system with known solutions, whereas for $t = 1$, $H(x,1) = F(x) = 0$ yields the solution of the original problems. The parameter t is called the homotopy parameter. If the functions $t \to x(t)$ and G are differentiable, then differentiating Eq. (3) with respect to t gives

$$\frac{\partial H}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial H}{\partial t} = 0, \quad x(0) = a, \quad 0 < t < 1, \tag{4}$$

which is a system of differential equations with the initial condition $x(0)$. More importantly, this actually shows that the solution of the algebraic equation presented in Eq. (3) can be transformed into an initial value problem of ordinary equations in the form of

$$\frac{\partial x}{\partial t} = -\frac{\partial H/\partial t}{\partial H/\partial x}. \tag{5}$$

It should be noted that, for different $G(x)$, there will be different integration paths in solving the differential equation (5). However, it can be verified that these paths will never have bifurcations and will never be infinite in length. Thus, in principle, they can be followed from one end to another, or, in the case of loops, from an arbitrary starting point back to the same point [13].

3. Homotopy solution of the eigenvalue problem for a locally reacting liner

The propagation of sound in a duct with uniform flow is governed by the convected wave equation

$$\nabla^2 p - \frac{1}{c_0^2}\frac{D_0^2 p}{Dt^2} = 0, \tag{6a}$$

where $D_0/Dt = (\partial/\partial t) + u_0(\partial/\partial z)$ and u_0 is the mean flow velocity in the axial direction, ∇^2 is the Laplacian operator and p is the acoustic pressure. To determine the sound attenuation in the lined duct shown in Fig. 1, the separation of variables can be applied to solve the equation. Furthermore, the application of the displacement continuity condition and momentum equation [1], gives

$$F(\gamma) = \gamma \tan(kb \cdot \gamma) - \mathrm{i}\beta_0 w^2 = 0, \tag{6b}$$

where $\gamma = k_r/k$, and

$$w = \frac{1 \mp Ma[1-(1-Ma^2)\gamma^2]^{1/2}}{1-Ma^2}, \tag{6c}$$

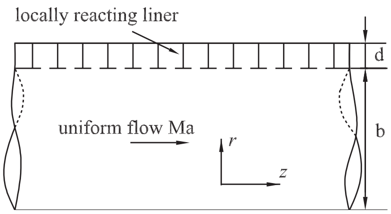

Fig. 1. The geometry of an acoustically locally reacting liner two-dimensional duct.

$$k_z/k = (-Ma \pm \sqrt{1-(1-Ma^2)\gamma^2})/(1-Ma^2), \tag{6d}$$

and β_0 is the specific admittance ratio, k is wavenumber, Ma is the Mach number of the mean flow in the duct. Besides, z is the duct axial coordinate and r is the transverse coordinate measured from the hard wall. k_{r_n} is the transverse wavenumber for the nth mode of propagation and k_{z_n} is the corresponding axial wavenumber.

In terms of the basic form of linear homotopy shown in Eq. (3), the two different forms of the homotopy equation can be constructed into

$$H(\gamma, t) = (1-t)\tan(kb\cdot\gamma) + tF(\gamma) = 0, \tag{7a}$$

$$H(\gamma, t) = (1-t)(\gamma-\gamma_0) + tF(\gamma) = 0, \tag{7b}$$

where γ_0 is the eigenvalue of hard wall. In Eq. (7a), $G(x) = \tan(kb\cdot\gamma_0)$, while in Eq. (7b), $G(x) = (\gamma-\gamma_0)$, γ_0 is the starting value at $t = 0$. As introduced in Eq. (5), the relevant differential equations can be arrived at

$$\frac{d\gamma}{dt} = -\frac{-\tan(kb\cdot\gamma) + F(\gamma)}{(1-t)kb\sec^2(kb\cdot\gamma) + t(\partial F/\partial\gamma)}, \tag{8a}$$

$$\frac{d\gamma}{dt} = -\frac{-(\gamma-\gamma_0) + F(\gamma)}{(1-t) + t(\partial F/\partial\gamma)}. \tag{8b}$$

As we mentioned in Section 2, different $G(x)$ can be used to form different homotopy equations. Obviously, Eqs. (8a) and (8b) are both nonlinear differential equations with the same initial value. Thus, the solution of the algebraic equation shown in Eqs. (7a) and (7b) has been transformed into an initial value problem of ODE. Naturally, various tools for the solution of ODE can be used to calculate the eigenvalue. On the other hand, it is noted that Eversman [1,2] derived the following ordinary equation by assuming that the eigenvalues are a function of some parameter, t and letting the admittance $\beta_0(t) = t\beta_{0f}$ vary from $0 \leqslant t \leqslant 1$

$$\frac{d\gamma}{dt} = \frac{iw^2\beta_{0f}}{\tan(kb\cdot\gamma) + kb\cdot\gamma\sec^2(kb\cdot\gamma) \mp 2i\beta_0(t)wMa\gamma/v^{1/2}}, \tag{8c}$$

where $v = 1-(1-Ma^2)(k_z/k)^2$. It is noted that for the case of plane wave, Eq. (8c) will become singular, since the starting value $\gamma = 0$. Eversman proposed to use small β_0 and γ for the first step of integration to avoid this problem. In addition, if we use Eqs. (8a) and (8b), there is no singularity for any starting values, and naturally it is more convenient to obtain more accurate results by numerically integrating the related ordinary equations. Table 1 shows an excellent comparison of the results from Eqs. (8a) and (8b) and those from Ref. [2]. As we have mentioned previously, using different homotopy equations will result in different integration paths, but the homotopy method guarantees that these paths will finally return to the same point. Fig. 2 shows such results using different homotopy equations.

Table 1
Comparison of results for (k_r/k) in a lined two-dimensional duct with uniform flow, $Ma = 0.5$, $kb = 1.0$, $\beta_0 = 0.72 - i0.42$

Mode	Eq. (8a)	Eq. (8b)	Eversman	Starting value
1+	1.141 + i0.217	1.141 + i0.217	1.142 + i0.217	0.0 + i0.0
2+	4.157 − i0.782	4.157 − i0.782	4.158 − i0.781	π + i0.0
3+	4.313 − i1.988	4.313 − i1.988	4.306 − i1.995	2π + i0.0
4+	7.857 − i0.479	7.916 − i0.619	7.857 − i0.478	3π + i0.0
5+	11.046 − i0.327	11.077 − i0.406	11.046 − i0.326	4π + i0.0
1−	0.523 + i0.353	0.523 + i0.353	0.523 + i0.353	0.0 + i0.0
2−	2.389 + i0.312	2.389 + i0.312	2.388 + i0.312	π + i0.0
3−	5.244 − i0.081	5.244 − i0.081	5.243 − i0.081	2π + i0.0
4−	8.214 − i0.165	8.214 − i0.165	8.214 − i0.165	3π + i0.0
5−	11.249 − i0.170	11.249 − i0.170	11.248 − i0.170	4π + i0.0

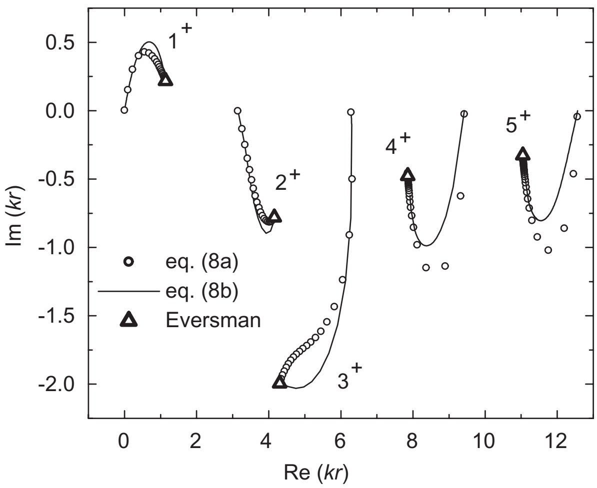

Fig. 2. The integration paths formed by using the different homotopy equations, $Ma = 0.5$, $kb = 1.0$, $\beta_0 = 0.72 - \mathrm{i}0.42$.

4. Homotopy solution of the eigenvalue problem for a non-locally reacting liner

4.1. Eigenvalue equation for a single-layer non-locally reacting liner

Consider in the polar cylindrical coordinate system (x, r, θ) a duct with rigid walls lined with an isotropic porous medium material. Also, we assume that the effect of the mean flow velocity in the porous medium is negligible, provided the resistivity of the material is not small. With these assumptions [5], the acoustic field in the porous medium is governed by

$$\rho_e \frac{\partial u_p}{\partial t} + \nabla p_p + \sigma u_p = 0, \tag{9}$$

$$\frac{\Omega}{\rho_0} \frac{\partial \rho_p}{\partial t} + \nabla u_p = 0, \tag{10}$$

$$p_p = c_e^2 \rho_p, \tag{11}$$

where c is the speed of sound, c_e is the effective speed of sound in the porous material, ρ_0 is density, $\rho_e = s\rho_0/\Omega$ is the effective density of the gas in the porous material, Ω is the porosity of the reacting liner material, σ is the resistivity of porous material, and s is a structure factor which is mainly dependent on the structural properties of the material and the internal friction of the gas. Obviously, propagation in the duct with a non-locally reacting liner can be expressed as superposition of acoustic modes of the form

$$p_n = A_n J_m(\alpha r) \mathrm{e}^{\mathrm{i}(\omega t - k_{z_n} z)}, \quad r < r_1, \tag{12a}$$

$$p_n = [B_n J_m(\beta r) + C_n Y_m(\beta r)] \mathrm{e}^{\mathrm{i}(\omega t - k_{z_n} z)}, \quad r_1 < r < r_2, \tag{12b}$$

where α is the eigenvalue of the non-locally reacting liner in the radial direction ($r \leqslant r_1$), and β is the eigenvalue in the radial direction ($r_1 \leqslant r \leqslant r_2$). By the application of the displacement continuity condition and momentum equation, the relevant relations can be written as

$$\left. \frac{\partial p}{\partial r} \right|_{r=r_2} = 0, \tag{13a}$$

$$\left(1 - Ma\frac{k_z}{k}\right)^{-2}\frac{\mathrm{d}p}{\mathrm{d}r}\bigg|_{r=r_1-0} = \zeta\frac{\mathrm{d}p}{\mathrm{d}r}\bigg|_{r=r_1+0}, \tag{13b}$$

$$[p]_{r=r_1-0}^{r=r_1+0} = \frac{Z}{\mathrm{i}\omega\rho_0}\frac{\mathrm{d}p}{\mathrm{d}r}\bigg|_{r=r_1+0}. \tag{13c}$$

And

$$k_z^2(1 - Ma^2) + 2Makk_z(\alpha^2 - k^2) = 0, \tag{14a}$$

$$k_z^2 + \beta^2 = \mu^2 = \left[k\frac{c}{c_e}\left(\frac{\Omega}{\zeta}\right)^{1/2}\right]^2, \tag{14b}$$

where

$$\zeta = \frac{\rho_0}{(\rho_e - \mathrm{i}(\sigma/\omega))}, \quad \mu = \left(\frac{\omega}{c_e}\right)\left(\frac{\Omega}{\zeta}\right)^{1/2}, \tag{15}$$

and Z is the specific acoustic impedance ratio. Substituting Eqs. (12) into Eqs. (13) yields

$$F(\alpha) = \left(1 - Ma\frac{k_z}{k}\right)^2\frac{J_m(\alpha r_1)}{\alpha J'_m(\alpha r_1)} + \frac{Z}{\mathrm{i}\omega\rho_0} - \frac{1}{\zeta\beta}\times\frac{J_m(\beta r_1)Y'_m(\beta r_2) - Y_m(\beta r_1)J'_m(\beta r_2)}{J'_m(\beta r_1)Y'_m(\beta r_2) - Y'_m(\beta r_1)J'_m(\beta r_2)} = 0. \tag{16}$$

When there is no flow in the duct, Eq. (16) is the same as Rienstra's equation [6]. However, up to now, it has not been solved numerically. We solve this equation by the following method. First, according to the eigenvalue equation derived as above, the relevant homotopy equation can be written as

$$H(k_r, t) = (1 - t)k_rJ'_m(k_{r0}r) + tF(k_r) = 0, \tag{17a}$$

$$H(k_r, t) = (1 - t)(k_r - k_{r0}) + tF(k_r) = 0. \tag{17b}$$

Then Eqs. (17a) and (17b) can be transformed into two ODEs with the same initial values, which are finally solved by the integration scheme. We have found that it is difficult to obtain reasonable results for a porous material liner without perforated plates by simply using Eqs. (17a) and (17b). To overcome this difficulty, we can assume that the parameter s, Ω and σ are also a function of t, satisfy $s(0) = 1$, $\Omega(0) = 1$, $\sigma(0) = 0$ for $t = 0$ and $s(1) = s_f$, $\Omega(1) = \Omega_f$, $\sigma(1) = \sigma_f$ for $t = 1$. Physically, the starting value ($t = 0$) and the final value ($t = 1$) correspond to no porous material in the liner and the real boundary condition with porous material, respectively, whereas it is important to set up an appropriate expression of $s(t)$, $\Omega(t)$ and $\sigma(t)$ for different liner configurations in order to obtain robust numerical results. In general, these parameters can be considered as linear or exponential functions of the homotopy parameter. As the first example, we consider a single-layer, non-locally reacting liner without a cover plate. In this case, Fig. 3 shows the comparison between the present calculation and Nayfeh's result [4], which was solved using the Newton–Raphson Scheme. Good agreement is obtained in terms of the trends of the results. The difference in the magnitude may be attributed to the effect of non-uniform flow considered in Nayfeh's model. The second example considers the liner without porous materials. The sound absorption mechanism originates from the vortex sound interactions [7,15]. With a bias flow Mach number of 0.03, Fig. 4 gives the frequency dependence of the radial eigenvalues and relevant sound attenuation with the frequency for the different modes.

4.2. Eigenvalue equation of double-layer non-locally reacting liners

The geometry of a double-layer acoustically non-locally lined circular duct with uniform flow is shown in Fig. 5. α, β_1, and β_2 denote, respectively, the eigenvalues of the radial direction in the two layers. Therefore, the sound propagation in the duct can be described as

$$p_n = A_nJ_m(\alpha_n r)\mathrm{e}^{\mathrm{i}(\omega t - k_{z_n}z)}, \quad r < r_1,$$

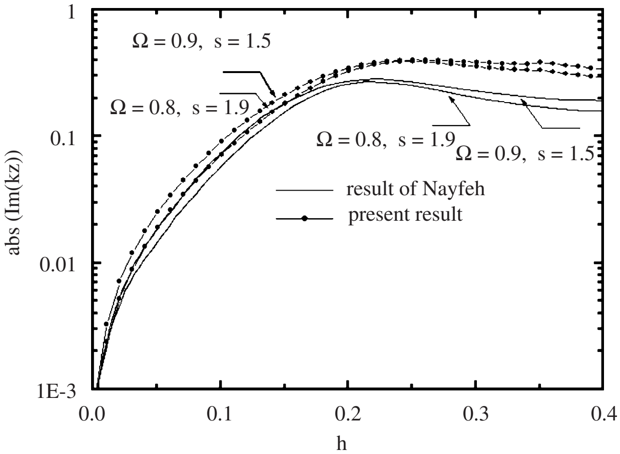

Fig. 3. Variation of the attenuation rate with liner thickness, $\sigma = 12.92$ and $\omega = 4$.

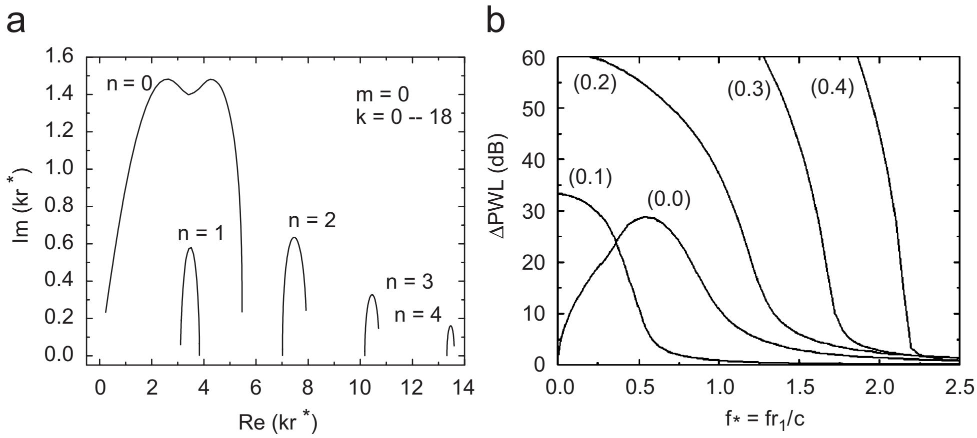

Fig. 4. Effect of frequency variation on the eigenvalues (a) and insertion loss (b), $r_1 = 1.0\,\mathrm{m}$, $h = 0.2$, $Ma = 0$, $L = 1.0$, $a = 3\,\mathrm{mm}$, $\phi = 5\%$, $Ma(U) = U/c_0 = 0.03$.

$$p_n = [B_{1n}J_m(\beta_{1n}r) + C_{1n}Y_m(\beta_{1n}r)]\mathrm{e}^{\mathrm{i}(\omega t - k_{z_n}z)}, \quad r_1 < r < r_2,$$

$$p_n = [B_{2n}J_m(\beta_{2n}r) + C_{2n}Y_m(\beta_{2n}r)]\mathrm{e}^{\mathrm{i}(\omega t - k_{z_n}z)}, \quad r_2 < r < r_3. \tag{18}$$

The boundary conditions of the double-layer non-locally reacting liner are

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_3} = 0, \tag{19a}$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_2^+} = \left.\frac{\partial p}{\partial r}\right|_{r=r_2^-}, \tag{19b}$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_2^+} = (p_{2^+} - p_{2^-})\eta_2|_{r=r_2}, \tag{19c}$$

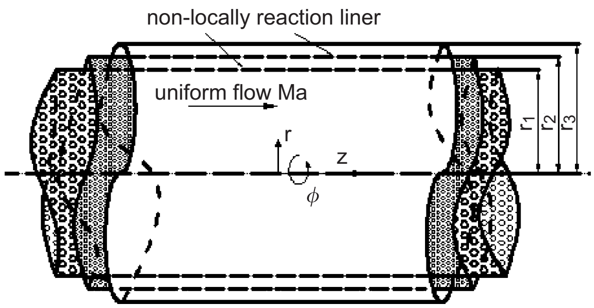

Fig. 5. A circular duct with double-layer, acoustically non-locally reacting liner circular duct.

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_1^+} = \left(1 - Ma\frac{k_z}{k}\right)^{-2} \left.\frac{\partial p}{\partial r}\right|_{r=r_1^-}, \tag{19d}$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_1^+} = (p_{1^+} - p_{1^-})\eta_1|_{r=r_1}. \tag{19e}$$

By imposing the corresponding boundary conditions, we can arrive at the following eigenvalue equation for the double-layer non-locally reacting liners

$$F(\alpha) = \frac{1}{\eta_1} + \left(1 - Ma\frac{k_z}{k}\right)^2 \frac{J_m(\alpha r_1)}{\alpha J_m'(\alpha r_1)} - \frac{1}{\beta}\frac{B_1 J_m(\beta r_1) + C_1 Y_m(\beta r_1)}{B_1 J_m'(\beta r_1) + C_1 Y_m'(\beta r_1)}, \tag{20}$$

where

$$\frac{B_1}{C_1} = -\frac{\eta_2 Y_m'(\beta r_3) f_1 + \beta Y_m'(\beta r_2) f_2}{\eta_2 J_m'(\beta r_3) f_1 + \beta J_m'(\beta r_2) f_2}, \tag{21}$$

$$f_1 = J_m(\beta r_2) Y_m'(\beta r_2) - J_m'(\beta r_2) Y_m(\beta r_2), \tag{22a}$$

$$f_2 = J_m'(\beta r_2) Y_m'(\beta r_2) - J_m'(\beta r_2) Y_m'(\beta r_2), \tag{22b}$$

and η is effective compliance [7]. Therefore, the homotopy equation can be expressed as

$$H(\alpha, t) = (1 - t) J_m'(\alpha r_1) + tF(\alpha) = 0. \tag{23}$$

The same procedure as that used for a single-layer non-locally reacting liner can be applied to solve the above homotopy equation. In order to verify the computational results from Eq. (20), we have made a comparison with recent experimental data [10]. In this experiment, $r_1 = 12.7\,\text{cm}$, $r_2 = 15.2\,\text{cm}$, $\phi_1 = 0.04$, $\phi_2 = 0.02$, $L = 17.8\,\text{cm}$, $a_1 = 3.3\,\text{mm}$, $a_2 = 2.7\,\text{mm}$ and $r_3 \gg r_1$. In fact, with the increase of r_3, the computation indicates that the absorption changes little. Fig. 6 shows that our model for the double non-locally reacting liners can give good agreement with the existing experimental results.

4.3. *Controllability of non-locally reacting liner by the bias flow*

As we know, if the eigenvalue of the softwall has been calculated, the mode-matching method [14] can be used robustly to predict the attenuation of multi-segmented liners. To investigate the possibility of controlling the sound attenuation of non-locally reacting liner through the bias flow, we consider a three-segmented liner, as shown in Fig. 7. The parameters ϕ and a represent the open area ratio and radius of the liner, respectively.

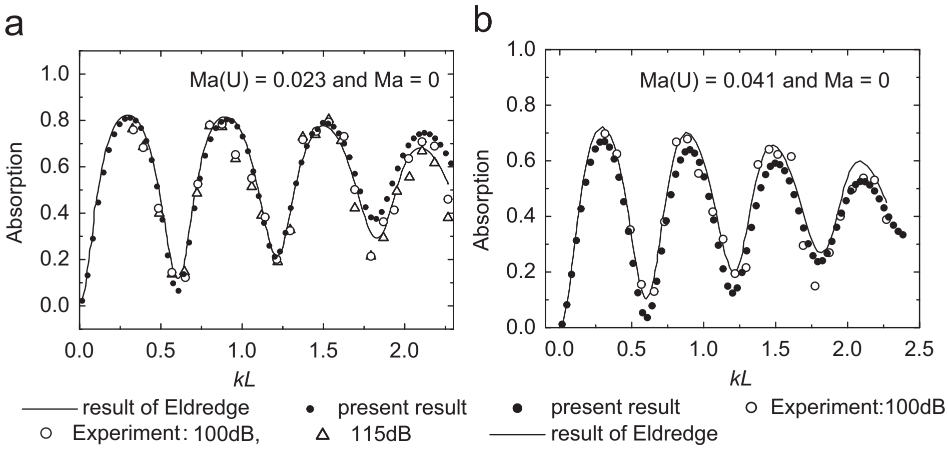

Fig. 6. Comparison with other results [10].

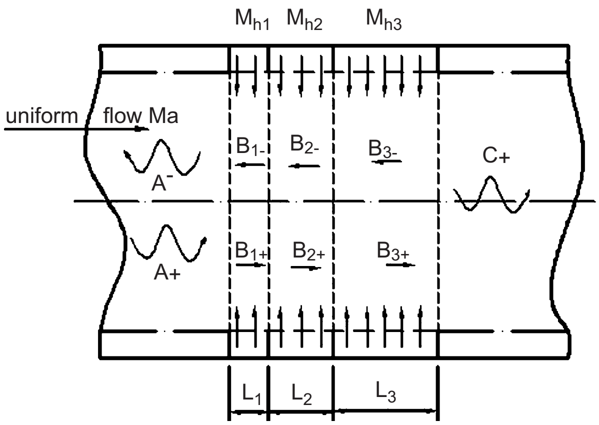

Fig. 7. The geometry of segmented non-locally reacting liners, $L_1 = 0.15\,\mathrm{m}$, $L_2 = 0.25\,\mathrm{m}$, $L_3 = 0.40\,\mathrm{m}$, $a = 2\,\mathrm{mm}$, $r_1 = 1.0\,\mathrm{m}$, $r_2 = 1.2\,\mathrm{m}$, $\phi = 4\%$.

For circumferential mode $m = 2$, Fig. 8(a) gives the results for sound attenuation with a uniform bias flow for each segment of the liner. We can see that the variation of the reduced frequency kL will correspond to different sound attenuation. Furthermore, if a different combination of the bias flow for each section is used, as shown in Fig. 8(b), we can achieve better sound attenuation, by making use of the sound wave cancelation between the interfaces with different impedance walls. In future work, we may further explore the potential of multi-segmented liners, building on the present and previous work [10,16,17].

5. Conclusions

This paper presents a unified algorithm to study the eigenvalue problem for a lined duct using the homotopy method. The analysis has been carefully benchmarked and validated against the existing work. The homotopy method developed in present study can provides an accurate and reliable means for sound-propagation computation. The approach not only overcomes the computational difficulties of the existing methods for locally reacting liners, but also provide a completely different way to calculate the eigenvalues of non-locally reacting liners. The latter is of great interest due to its potential application for future advanced liners. Finally, a model multi-segmented, non-locally reacting liners is treated to explore the possibility of controlling sound

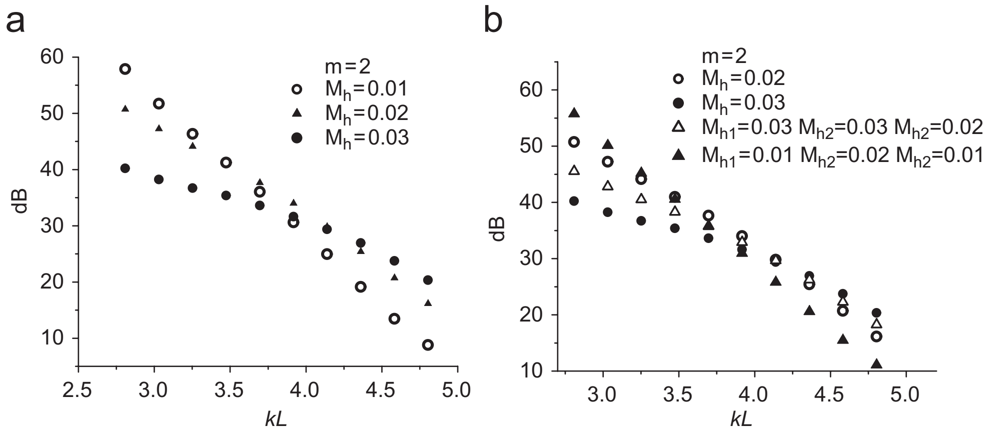

Fig. 8. Effect of the variations of frequency and bias flow on the attenuation.

attenuation through a bias flow. Simulation results indicated that optimal sound attenuation can be achieved by adjusting the bias flow of each segment.

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